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DAVIDSON LABORATORY

Report 933

WAVE-RESISTANCE REDUCTION OF NEAR-SURFACE BODIES

by

King Eng and Pung N. Hu

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John Breslin
Dr. John Breslin
Director

ABSTRACT

An analytical study of the wave-resistance characteristics of near-surface bodies was conducted to determine 1) for a given length and displacement, what changes in body-surface geometry are necessary to cause wave-resistance reduction, and 2) how geometrical change affects the wave-resistance behavior with Froude number and submergence depth. The general wave-resistance expression for a perturbed ellipsoid with the constraints of constant displacement and length is formulated. A digital computer solution of this variational problem is obtained for the case of the spheroid due to available computer-size limitations.

The effects of fineness ratio and submergence-to-length ratio on the Froude number behavior of the wave resistance for a range of perturbations is demonstrated. Substantial reduction in wave resistance is possible for all Froude numbers above and slightly below the optimum Froude number for a particular perturbation distribution. For Froude numbers lower than approximately 10% below the optimum Froude number, a large increase in wave-resistance coefficient may be obtained depending upon the perturbation used. Since this generally occurs at low Froude numbers, the actual increase in total resistance experienced for perturbations yielding acceptable geometrical changes should be quite acceptable. Depending upon the optimum Froude number, the geometrical changes required for wave-resistance reduction fall into two classes: 1) midsection bulge with finer bow and stern for Froude numbers below 0.32; and 2) above 0.32 Froude number a midsection pinch with bulging bow and stern.

NOMENCLATURE

a_1, a_2, a_3	semi-axes of an ellipsoid
A, A_m	perturbation parameter
D	diameter of spheroid
e, $\bar{e}$	eccentricity of ellipse
$F = \frac{U}{gL}$	Froude number
g	acceleration of gravity
k_o	$\frac{g}{U^2}$
k_1	longitudinal added mass coefficient
r_o, r, r_1	radius
R_o, R	wave resistance
R_o', R', R_m'	wave-resistance coefficient
$-U$	constant uniform stream velocity
x_1, x_2, x_3	rectangular coordinates
ξ_1, ξ_2, ξ_3	rectangular coordinates
$\phi, \phi_o, \phi_1, \Phi$	velocity potential
μ, μ_o, μ_1	strength of doublets with axes in the positive x_1-direction
$\bar{\mu}$	normalized strength of doublets with axes in the positive x_1-direction
ρ	mass density of fluid

Subscript m denotes optimum value

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INTRODUCTION

As an integral part of the research program on high-speed ship forms at Davidson Laboratory, an analytical investigation into the reduction of the wave resistance of a submerged body moving close to the water surface was conducted. The major problems of interest in the investigation were:

(1) For a given volume and length, how should the surface geometry be changed in order to cause a reduction in wave resistance?

(2) How does the geometrical change affect the wave-resistance behavior with Froude number and depth of immersion?

The linearized theory of wave resistance for bodies moving near the surface has been well established by Michell, Havelock, Lunde, etc. These theories impose a linearized free-surface boundary condition on the velocity potential. The wave-resistance expression is an integral with a quadratic integrand consisting either of functions that define the shape of the hull, or functions that define some type of hydrodynamic singularities by which the hull is generated. The latter type of integrand, being mathematically more tractable, is used in this study. The purpose here is to find a hull geometry of minimum wave resistance. Therefore, the investigation becomes a variational problem, and it is apparent that the variation should be in the hydrodynamic singularities.

Weinblum[1] treated such a minimum problem by considering a family of hull curves whose doublet distribution was expressed by polynomials having several arbitrary parameters. His result shows that, for a given Froude number and immersion depth, the doublet distribution and its

corresponding wave resistance can be evaluated in terms of a table of functions. However, no comparison can be made with his results because the hull displacement is not constrained.

The general case of a perturbed ellipsoid is considered in this analysis. It is approached as a variational problem with constant displacement as a subsidiary condition. The ellipsoid is represented by doublets distributed over the confocal ellipse. This doublet distribution is then perturbed such that the perturbation will have no influence on the volume of the ellipsoid, and will produce a new hull with less wave resistance.

THEORY

Throughout the discussion, the axes x_1, x_2 and x_3 of a right-hand Cartesian coordinate system are fixed on the moving body. The origin O has been taken at the geometric center of the body with Ox_1 parallel to the direction of motion and Ox_3 vertically upward. The fluid is assumed to be incompressible and inviscid. The motion is irrotational and characterized by a velocity potential ϕ which defines the fluid velocity $\bar{q}$ by $\bar{q} = -\nabla\phi$. The wave height on the free surface is taken to be small in comparison to the wave length.

Consider an ellipsoid with semi-axes $a_1>a_2>a_3$ moving in an infinite fluid at a constant speed U along the x_1-direction. The velocity potential which describes the absolute motion of the fluid is given by[2]

$$\phi_0(x_1,x_2,x_3) = \iint\limits_{\xi_3=0} \mu_0(\xi_1,\xi_2) \frac{\partial}{\partial\xi_1} \left(\frac{1}{r}\right) d\xi_1 d\xi_2 \tag{1}$$

where

$$\mu_o(\xi_1,\xi_2) = \frac{UD_1}{\pi}\left[1 - (\frac{\xi_1}{a_1 e})^2 - (\frac{\xi_2}{a_2 \bar{e}})^2\right]^{1/2} \tag{2}$$

$$e^2 = 1 - (\frac{a_3}{a_1})^2 \tag{3}$$

$$\bar{e}^2 = 1 - (\frac{a_3}{a_2})^2 \tag{4}$$

$$D_1 = \frac{a_3}{(2-\alpha_1)e\bar{e}} \tag{5}$$

$$\alpha_1 = a_1 a_2 a_3 \int_0^\infty \frac{d\lambda}{(a_1^2+\lambda)\sqrt{(a_1^2+\lambda)(a_2^2+\lambda)(a_3^2+\lambda)}} \tag{6}$$

$$r^2 = (x_1-\xi_1)^2 + (x_2-\xi_2)^2 + (x_3-\xi_3)^2 \tag{7}$$

The surface integral in eq. 1 is taken over the confocal ellipse

$$(\frac{\xi_1}{a_1 e})^2 + (\frac{\xi_2}{a_2 \bar{e}})^2 = 1 \tag{8}$$

on the plane $\xi_3 = 0$. Let the ellipsoid be perturbed such that the perturbation can be represented by a doublet distribution $\mu_1(\xi_1,\xi_2)$ in addition to the original doublet distribution $\mu_o(\xi_1,\xi_2)$. $\mu_1(\xi_1,\xi_2)$ is bounded by the same confocal ellipse given by eq. 8. The perturbation potential is then

$$\phi_1(x_1,x_2,x_3) = \iint_{\xi_3=0} \mu_1(\xi_1,\xi_2) \frac{\partial}{\partial\xi_1}(\frac{1}{r})\, d\xi_1 d\xi_2 \tag{9}$$

The resultant potential for the perturbed ellipsoid becomes

$$\phi(x_1,x_2,x_3) = \iint_{\xi_3=0} \mu(\xi_1,\xi_2) \frac{\partial}{\partial\xi_1}(\frac{1}{r})\, d\xi_1 d\xi_2 \tag{10a}$$

where $\phi(x_1,x_2,x_3) = \phi_o(x_1,x_2,x_3) + \phi_1(x_1,x_2,x_3)$ (10b)

and $\mu(\xi_1,\xi_2) = \mu_o(\xi_1,\xi_2) + \mu_1(\xi_1,\xi_2)$ (10c)

is the resultant doublet distribution. It is required that the volume remains constant upon this perturbation; therefore,*

$$\iint\limits_{\xi_3=0} \mu_1(\xi_1,\xi_2)\, d\xi_1 d\xi_2 = 0 \tag{11}$$

By letting $\mu_1(\xi_1,\xi_2) = \mu_o(\xi_1,\xi_2)Q$, where $Q = Q(\xi_1,\xi_2)$ is an arbitrary function to be chosen later, eq. 10 becomes

$$\phi(x_1,x_2,x_3) = \frac{UD_1}{\pi}\iint\limits_{\xi_3=0}\left[1-\left(\frac{\xi_1}{a_1 e}\right)^2 - \left(\frac{\xi_2}{a_2 e}\right)^2\right]^{\frac{1}{2}} (1+Q)\frac{\partial}{\partial\xi_1}\left(\frac{1}{r}\right) d\xi_1 d\xi_2 \tag{12}$$

* According to Taylor's added mass theorem,
$U\rho V(1+k_1) = 4\pi\rho\iint(\mu_o+\mu_1)d\xi_1 d\xi_2 = U\rho V_o(1+k_1^o) + 4\pi\rho\iint\mu_1 d\xi_1 d\xi_2$
where k_1, V and k_1^o, V_o are longitudinal added mass coefficient and volume of the perturbed and unperturbed ellipsoid, respectively.

Then $V(1+k_1) - V_o(1+k_1^o) = \frac{4\pi}{U}\iint\mu_1 d\xi_1 d\xi_2 = 0$ by eq. 11

or $V = V_o\left(\frac{1+k_1^o}{1+k_1}\right)$

For elongated bodies of approximately same length and L/D, k_1 and k_1^o are small in comparison to unity; also they are of the same order of magnitude, i.e, $k_1 - k_1^o$ is very small. Therefore,

$$\frac{1+k_1^o}{1+k_1} \doteq 1 \text{ or } V \doteq V_o$$

When the body is moving below a free surface, the velocity potential must satisfy the linearized boundary condition

$$\frac{\partial^2\Phi}{\partial x_1^2} + k_o \frac{\partial\Phi}{\partial x_3} = 0 \tag{13}$$

where $k_o = \frac{g}{U^2}$

on the free surface $(x_3 = f)$.

The Green's function, satisfying this condition (eq. 13), is given by [5,8]

$$G(x_1,x_2,x_3;\ \xi_1,\xi_2,\xi_3) = \frac{1}{r} - \frac{1}{r_1}$$

$$- \frac{4k_o}{\pi} \text{Re} \int_0^{\frac{\pi}{2}}\int_0^{\infty} \sec^2\theta \cdot e^{k[(x_3+\xi_3-2f)+i(x_1-\xi_1)\cos\theta]} \frac{\cos[k(x_2-\xi_2)\sin\theta]}{k-k_o\sec^2\theta} dk d\theta \tag{14}$$

where $r_1^2 = (x_1-\xi_1)^2 + (x_2-\xi_2)^2 + (x_3+\xi_3-2f)^2$

Physically, the Green's function represents the velocity potential of a source moving at a depth f below the free surface. Therefore, to a first-order approximation, the velocity potential of the body, represented by the doublet distribution $\mu(\xi_1,\xi_2)$, moving below a free surface may be expressed as

$$\Phi = \Phi(x_1,x_2,x_3) = \iint_{\xi_3=0} \mu(\xi_1,\xi_2)\ \frac{\partial G}{\partial \xi_1} d\xi_1 d\xi_2 \tag{15}$$

Substituting eq. 14 into eq. 15, the potential Φ for $(x_1-\xi_1)>0$ and $(x_1-\xi_1)<0$ becomes, respectively,

$$\Phi = \iint \mu(\xi_1,\xi_2)(x_1-\xi_1)\left(\frac{1}{r^3} - \frac{1}{r_1^3}\right)d\xi_1 d\xi_2$$

$$- \frac{4k_o}{\pi}\, \mathrm{Im} \iint \mu(\xi_1,\xi_2) \int_0^{\frac{\pi}{2}} \int_0^{1\infty} \frac{k\sec\theta}{k-k_o\sec^2\theta} \cos\left[k(x_2-\xi_2)\sin\theta\right]$$

$$e^{k[(x_3-2f) + i(x_1-\xi_1)\cos\theta]}\, d\xi_1 d\xi_2 dk d\theta$$

and $$\Phi = \iint \mu(\xi_1,\xi_2)\ (x_1-\xi_1)\left(\frac{1}{r^3} - \frac{1}{r_1^3}\right) d\xi_1 d\xi_2$$

$$- \frac{4k_o}{\pi}\, \mathrm{Im} \iint \mu(\xi_1,\xi_2) \int_0^{\frac{\pi}{2}} \int_0^{1\infty} \frac{k\sec\theta}{k+k_o\sec^2\theta} \cos\left[k(x_2-\xi_2)\sin\theta\right]$$

$$e^{-k[(x_3-2f)\ i\ |x_1-\xi_1|\ \cos\theta]}\, d\xi_1 d\xi_2 dk d\theta$$

$$+ 8k_o^2 \int_0^{\frac{\pi}{2}} \iint \mu(\xi_1,\xi_2)\ \cos\left[k_o(x_1-\xi_1)\sec\theta\right]\cdot \cos\left[k_o(x_2-\xi_2)\right.$$

$$\left.\sec^2\theta\ \sin\theta\right] \sec^3\theta\quad e^{k_o(x_3-2f)\sec^2\theta}\ d\xi_1 d\xi_2 d\theta$$

From ref. 6, the wave-resistance expression derived from a consideration of the energy expended in the production of waves is given as

$$R = \frac{\rho}{2k_o}\int_{-\infty}^{\infty}\left[\left(\frac{\partial\Phi}{\partial x_1}\right)^2 - \Phi\,\frac{\partial^2\Phi}{\partial x_1^2}\right]_{\substack{x_3=f\\ x_1\to-\infty}} dx_2 - \frac{\rho}{2}\int_{-\infty}^{f}\int_{-\infty}^{\infty}\left[\left(\frac{\partial\Phi}{\partial x_1}\right)^2 - \Phi\frac{\partial^2\Phi}{\partial x_1^2}\right]_{x_1\to-\infty} dx_2 dx_3 \tag{16}$$

From ref. 5, the velocity potential of Φ at $x_1\to-\infty$ can be approximated to the form

$$\Phi\Big|_{x_1\to-\infty} = 8k_o^2\int_0^{\pi/2}\iint\mu(\xi_1,\xi_2)\cos q_1\left(\frac{x_1-\xi_1}{a_1 e}\right)\cos q_2\left(\frac{x_2-\xi_2}{a_2\bar{e}}\right)$$

$$\sec^3\theta \cdot e^{k_o(x_3-2f)\sec^2\theta}\, d\xi_1 d\xi_2 d\theta \tag{17}$$

where $q_1 = k_o a_1 e \sec\theta$; $q_2 = k_o a_2 \bar{e} \sec^2\theta \sin\theta$.

Substituting eq. 17 into eq. 16, one gets:

$$R = 16\pi\rho k_o^4\int_0^{\frac{\pi}{2}}(P_1^2+P_2^2+P_3^2+P_4^2)\sec^5\theta e^{-2k_o f\sec^2\theta}\, d\theta \tag{18}$$

where

$$P_1 = \iint\mu(\xi_1,\xi_2)\cos q_1\left(\frac{\xi_1}{a_1 e}\right)\cos q_2\left(\frac{\xi_2}{a_2\bar{e}}\right)d\xi_1 d\xi_2 \tag{19a}$$

$$P_2 = \iint\mu(\xi_1,\xi_2)\sin q_1\left(\frac{\xi_1}{a_1 e}\right)\cos q_2\left(\frac{\xi_2}{a_2\bar{e}}\right)d\xi_1 d\xi_2 \tag{19b}$$

$$P_3 = \iint\mu(\xi_1,\xi_2)\sin q_1\left(\frac{\xi_1}{a_1 e}\right)\sin q_2\left(\frac{\xi_2}{a_2\bar{e}}\right)d\xi_1 d\xi_2 \tag{19c}$$

$$P_4 = \iint\mu(\xi_1,\xi_2)\cos q_1\left(\frac{\xi_1}{a_1 e}\right)\sin q_2\left(\frac{\xi_2}{a_2\bar{e}}\right)d\xi_1 d\xi_2 \tag{19d}$$

The P_1^2 terms in eq. 18 are all positive definite quantities. Therefore, each P_1^2 term will contribute to wave resistance. However, if $\mu(\xi_1, \xi_2)$ is an even function with respect to ξ_1 and ξ_2 , i.e., $\mu(\xi_1,\xi_2) = \mu(-\xi_1,-\xi_2)$, then P_2, P_3 and P_4 vanish identically which to some extent reduces the resistance R. As a result, doubly-symmetric bodies are better forms as far as wave resistance is concerned.

Consequently, the expression of the doublet distribution takes the form:

$$\mu(\xi_1,\xi_2) = \frac{UD_1}{\pi}\left[1 - \left(\frac{\xi_1}{a_1 e}\right)^2 - \left(\frac{\xi_2}{a_2 \bar{e}}\right)^2\right]^{1/2}(1 + Q)$$

Q will be chosen as an even function with respect to ξ_1 and ξ_2. Obviously, the choice of Q is not unique under these constraints. For the sake of mathematical simplicity, the choice of Q for the present study is

$$Q(\xi_1,\xi_2) = - A \cos \lambda\left(\frac{\xi_1}{a_1 e}\right) \cos \nu\left(\frac{\xi_2}{a_2 \bar{e}}\right) \tag{20}$$

where A, λ and ν are arbitrary parameters to be determined. The doublet distribution expression now takes the form

$$\mu(\xi_1,\xi_2) = \frac{UD_1}{\pi}\left[1 - \left(\frac{\xi_1}{a_1 e}\right)^2 - \left(\frac{\xi_2}{a_2 \bar{e}}\right)^2\right]^{1/2}\left[1 - A \cos \lambda\left(\frac{\xi_1}{a_1 e}\right)\cos\nu\left(\frac{\xi_2}{a_2 \bar{e}}\right)\right] \tag{21}$$

Substituting eq. 20 into the constraint equation (eq. 11), one obtains

$$-\left(\frac{UD_1}{\pi}\right) A \iint \left[1-\left(\frac{\xi_1}{a_1 e}\right)^2-\left(\frac{\xi_2}{a_2 \bar{e}}\right)^2\right]^{1/2} \cos \lambda\left(\frac{\xi_1}{a_1 e}\right)\cos \nu\left(\frac{\xi_2}{a_2 \bar{e}}\right) d\xi_1 d\xi_2 = 0$$

which can be satisfied if (see Appendix)

$$A\left(\sqrt{\lambda^2+\nu^2}\right)^{-3/2} J_{3/2}\left(\sqrt{\lambda^2+\nu^2}\right) = 0$$

For nontrivial solution of A, λ and ν, one has

$$\sqrt{\lambda^2 + \nu^2} = \tan\sqrt{\lambda^2 + \nu^2} \tag{22}$$

Substituting eq. 21 into eqs. 19a through 19d, the results are (see Appendix)

$$P_1 = U\bar{D}_1 (\psi_0 - A\psi) \tag{23}$$

$$P_2 = 0,\ P_3 = 0,\ \text{and}\ P_4 = 0 \tag{24}$$

where

$$\psi_o = 2\sqrt{\frac{\pi}{2}}\ \frac{J_{3/2}(\sqrt{q_1^2 + q_2^2})}{(\sqrt{q_1^2 + q_2^2})^{3/2}}\ ;\ \bar{D}_1 = \frac{a_1 a_2 a_3}{(2 - \alpha_1)}$$

$$\psi = \frac{1}{2}\sqrt{\frac{\pi}{2}}\left\{\frac{J_{3/2}(\sqrt{(\lambda-q_1)^2+(\nu-q_2)^2})}{(\sqrt{(\lambda-q_1)^2+(\nu-q_2)^2})^{3/2}} + \frac{J_{3/2}(\sqrt{(\lambda-q_1)^2+(\nu+q_2)^2})}{(\sqrt{(\lambda-q_1)^2+(\nu+q_2)^2})^{3/2}} + \right.$$

$$\left.\frac{J_{3/2}(\sqrt{(\lambda+q_1)^2+(\nu-q_2)^2})}{(\sqrt{(\lambda+q_1)^2+(\nu-q_2)^2})^{3/2}} + \frac{J_{3/2}(\sqrt{(\lambda+q_1)^2+(\nu+q_2)^2})}{(\sqrt{(\lambda+q_1)^2+(\nu+q_2)^2})^{3/2}}\right\}$$

and $q_1 = k_o a_1 e \sec\theta$; $q_2 = k_o a_2 \bar{e} \sec^2\theta \sin\theta$

Substituting eq. 23 into eq. 18, R becomes

$$R = 16\pi\rho k_o^4(U\bar{D}_1)^2 \int_0^{\frac{\pi}{2}} (\psi_o - A\psi)^2\, g(\theta)\, d\theta \tag{25}$$

where $g(\theta) = \sec^5\theta e^{-k_o f \sec^2\theta}$

Let $L = 2a_1$, $F^2 = \frac{U^2}{gL} = \frac{1}{2k_o a_1}$, and $R' = \frac{R}{\frac{1}{2}\rho U^2 L^2}$

where F is the Froude number, and R' is the wave resistance coefficient. Then

$$R' = K\int_0^{\frac{\pi}{2}} (\psi_o - A\psi)^2\, g(\theta)\, d\theta \tag{26}$$

where $K = \frac{\pi}{2}\left(\frac{1 - e^2}{2 - \alpha_1}\right)\frac{1}{(1-\bar{e}^2)}\ \frac{1}{F^8}$

Denoting

$$R_o = K \int_0^{\frac{\pi}{2}} \psi_o^2 \; g(\theta) \; d\theta \tag{27a}$$

$$R_1 = K \int_0^{\frac{\pi}{2}} \psi\psi_o \; g(\theta) \; d\theta \tag{27b}$$

and $$R_2 = K \int_0^{\frac{\pi}{2}} \psi^2 \; g(\theta) \; d\theta \tag{27c}$$

R' can be written in terms of a perturbation parameter A* as follows:

$$R'(A) = R_o - 2AR_1 + A^2R_2 \tag{28}$$

The first and second variations of R'(A) are given, respectively, as

$$\delta R'(A) = 2(-R_1 + AR_2)\;\delta A \tag{29}$$

and

$$\delta^2 R'(A) = 2R_2(\delta A)^2 \tag{30}$$

since R_2 is positive definite as seen from eq. 27c, eq. 30 shows that if R'(A) has an extremal, it is a minimum. The minimum of R' occurs at

$$A = A_m = \frac{R_1}{R_2}^{**} \tag{31}$$

and has a value

$$R'(A_m) = R'_m = R_o - \frac{R_1^2}{R_2} \tag{32}$$

*R is also a function of λ and ν, but the major parameter which reduces wave resistance is the parameter A.

**Since $R_1 = R_1(\lambda,\nu)$ and $R_2 = R_2(\lambda,\nu)$, therefore $A_m = A_m(\lambda,\nu) = \frac{R_1(\lambda,\nu)}{R_2(\lambda,\nu)}$, then for various values of (λ,ν) that satisfy eq. 31 and the constraint condition eq. 22, a family of hull forms of similar wave-resistance characteristics will result.

Since it is not possible to obtain a closed form solution of R_0, R_1 and R_2, the problem solution requires the use of a large capacity digital computer. Due to the limited capacity and speed of Stevens Institute's 1620 IBM computer, numerical results are presented only for the general case of a perturbed spheroid.

The equations of a perturbed spheroid can be obtained by taking limits of the perturbed ellipsoid equations, i.e., let $n = 0$, $\xi_2 \longrightarrow 0$ and $\bar{e} \to 0$. The spheroid equations then become for doublet strength:

$$\mu(\xi) = \frac{U(a_1 e)^2}{2\left[\frac{2e}{1 - e^2} - \ln\left(\frac{1 + e}{1 - e}\right)\right]}(1 - \xi^2)(1 - A \cos \lambda\xi) \quad (33)$$

where $\xi = \frac{\xi_1}{a_1 e}$ and $-1 \leq \xi \leq 1$,

and for the potential in an infinite fluid:

$$\Phi(\xi) = \int_{-1}^{1} \mu(\xi) \frac{\partial}{\partial\xi}\left(\frac{1}{r}\right) d\xi \quad (34)$$

where $r^2 = (x_1 - a_1 e\xi)^2 + y^2$; $y^2 = x_2^2 + x_3^2$.

The characteristic equation resulting from the constant volume constraint reduces to:

$$\lambda = \tan \lambda \ . \quad (35)$$

The expressions R_0, R_1, R_2, R', A_m and R'_m remain unchanged, but K, ψ_0 and ψ are different.

$$R_0 = K \int_0^{\frac{\pi}{2}} \psi_0^2 \, g(\theta) \, d\theta \quad (36a)$$

$$R_1 = K \int_0^{\frac{\pi}{2}} \psi \, \psi_0 \, g(\theta) \, d\theta \quad (36b)$$

$$R_2 = K \int_0^{\frac{\pi}{2}} \psi^2 \; g(\theta) \; d\,\theta \tag{36c}$$

$$R' = R_o - 2AR_1 + A^2R_2 \tag{37}$$

$$A_m = \frac{R_1}{R_2} \tag{38}$$

$$R_m' = R_o - \frac{R_1^2}{R_2} \tag{39}$$

$$K = \frac{\pi}{2}\left\{\frac{e^3}{F^4\left[\frac{2e}{1 - e^2} - \ell n\left(\frac{1 + e}{1 - e}\right)\right]}\right\}^2$$

$$\psi_o = \frac{2}{q_1^3}\,(\sin q_1 - q_1 \cos q_1)$$

$$\psi = \frac{\sin(\lambda-q_1) - (\lambda-q_1)\cos(\lambda-q_1)}{(\lambda-q_1)^3} + \frac{\sin(\lambda+q_1) - (\lambda+q_1)\cos(\lambda+q_1)}{(\lambda+q_1)^3}$$

where $q_1 = \dfrac{e}{2F^2} \sec\,\theta$.

GEOMETRY OF PERTURBED SPHEROID*

The doublet distribution between foci

$$\mu_o(\xi) = \frac{Ua_1^2e^2(1 - \xi^2)}{2\left[\frac{2e}{1-e^2} - \ell n\left(\frac{1+e}{1-e}\right)\right]}, \quad \xi = \frac{x}{a_1 e} \tag{40}$$

is the image of the uniform stream -U within the spheroid:

$$r_o^2 = a_1^2(1 - e^2)\;(1 - e^2\xi^2) \tag{41}$$

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* This method is suggested by Professor L. Landweber, from the State University of Iowa.

An increment in the doublet distribution $\Delta\mu$ will produce a change in the ordinate of the spheroid which, it will be assumed, is given by modified Munk's formula:

$$\Delta\mu = \frac{1 + k_1}{4} U\Delta(r^2) \tag{42}$$

where k_1 is the added mass coefficient of the spheroid given by[12]

$$\frac{2}{1+k_1} = \frac{1-e^2}{e^3}\left[\frac{2e}{1-e^2} - \ln\left(\frac{1+e}{1-e}\right)\right] \tag{43}$$

Taking $\Delta\mu = \mu_1 = -A\mu_o(\xi)\cos\lambda\xi$, together with eqs. 40, 42 and 43:

$$\Delta(r^2) = -\frac{Aa_1^2(1-e^2)}{e}(1-\xi^2)\cos\lambda\xi \tag{44}$$

combine eqs. 41 and 44 and get:

$$r^2(\xi) = r_o^2 + \delta(r^2) = a_1^2(1-e^2)\left[1-e^2\xi^2-\frac{A}{e}(1-\xi^2)\cos\lambda\xi\right] \tag{45}$$

$$r(x) = \left(\frac{D}{L}\right)\sqrt{1-\left(\frac{x}{a_1}\right)^2-\frac{A}{e^3}\left[e^2-\left(\frac{x}{a_1}\right)^2\right]\cos\frac{\lambda}{e}\left(\frac{x}{a_1}\right)} \tag{46}$$

where $r(x)$ is the radius of the perturbed spheroid along the x-axis

$\left(\frac{D}{L}\right)$ is the slenderness ration of the undisturbed spheroid.

RESULTS AND ANALYSIS

The optimum perturbation parameter A_m is plotted versus Froude number in Fig. 1. At Froude numbers below .30, A_m is almost independent of both slenderness and immersion ratio. For Froude numbers $.28 < F < .30$, A_m shows little change and has a value of approximately $A_m = -.20$. For $F > .30$, A_m

varies significantly with depth, f/L, but varies very little with fineness ratio, D/L. This result is expected because at infinite depth, there is no wave resistance.

By examining the doublet distribution, one can obtain a good indication of what the approximate hull form will be like. Therefore, from eq. 33, one may conclude that:

1) for $A < 0$, the hull forms bulge out at the midsection and are narrow at the ends,

2) for $0 < A < 1$, the hull forms neck in at the midsection and bulge out at the ends,

3) for $A > 1$, the hull form may be imaginary.

There is no clear-cut dividing line as to what type of body geometry a hull may have and still be considered reasonable. However, the perturbed bodies with $A < 0.5$ generated from a spheroid could easily be considered reasonable forms.

The fact that the body geometry for minimum wave resistance varies with Froude number and immersion ratio makes it apparent that there is no single hull that can have minimum wave resistance over a range of Froude numbers and submergence depths. However, from Fig. 1 and eq. 33, for arbitrary values of A ranging from $-.20 < A < 1$, there is associated a hull form which will have a minimum at some F and f/L. For example: for $A = -.20$, the minimum will occur between $F = .28$ and .30. With this in mind, $A = -.20$, .25, .50 and .75 were chosen to illustrate the results of this analysis. The Froude number and submergence ratio corresponding to the minimum for the above perturbation parameters are shown in Fig. 2. The associated normalized doublet distributions and the approximate hull form are shown respectively in Figs. 3 and 4.

Figures 5 and 6, respectively, show the wave-resistance variation with Froude number and immersion ratio. As

would be expected, the wave resistance over the entire Froude-number range is affected by the geometrical change resulting from the perturbation. For $A > 0$ the wave-resistance coefficient, in general, has a reduction at high Froude-number values, but shows considerable increase at low values, especially when the body moves toward the free surface. Also, for $A < 0$, the wave-resistance coefficient reduces at low but increases at high Froude numbers.

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APPENDIX

Evaluate the surface integral of the form

$$I = \iint \sqrt{1 - (\frac{x}{a})^2 - (\frac{y}{b})^2}\ \cos\alpha x\ \cos\beta y dx dy$$

over the surface of an ellipse

$$(\frac{x}{a})^2 + (\frac{y}{b})^2 = 1$$

where α and β are arbitrary constants.

Let $x = a\sin\phi\cos\theta$ and $y = b\cos\phi$, then the element surface becomes $ds = ab\sin^2\phi\sin\theta d\phi d\theta$.

Upon change of variables, the integral I can be written as

$$I = ab\int_0^\pi\int_0^\pi \sin^3\phi\sin^2\theta\ \cos\left[(a\alpha\sin\phi)\cos\theta\right]\cos(b\beta\cos\phi)\,d\theta d\phi\ .$$

Denoting $\gamma = a\alpha\sin\phi$, and integrating with respect to θ:

$$P(\gamma) = \int_0^\pi \sin^2\theta\ \cos(\gamma\cos\theta)\,d\theta$$

$$= -\frac{1}{\gamma}\int_0^\pi \sin\theta\cos(\gamma\cos\theta)\ d(\gamma\cos\theta)\ .$$

Integrating $P(\gamma)$ by parts, we obtain

$$P(\gamma) = \frac{1}{\gamma}\int_0^\pi \cos\theta\ \sin(\gamma\cos\theta)\,d\theta = \frac{\pi}{\gamma}\ J_1(\gamma)\ .$$

Substitute $P(\gamma)$ back into I; then I becomes

$$I = \frac{\pi b}{\alpha}\int_0^\pi \sin^2\phi J_1(a\alpha\sin\phi)\cos(b\beta\cos\phi)\,d\phi$$

but $\cos\delta = \sqrt{\frac{\pi\delta}{2}}\ J_{-\frac{1}{2}}(\delta)$

therefore

$$I = \frac{\pi\sqrt{2\pi b\beta}}{\alpha}\int_0^{\pi} J_1(\alpha a\sin\phi)\ J_{-\frac{1}{2}}(b\beta\cos\phi)\ \cos^{\frac{1}{2}}\phi\ \sin^2\phi\, d\phi\ .$$

From ref. 13, under Sonine's second finite integral, the expression I is of the form

$$I = (\pi\sqrt{2\pi}ab)\frac{J_{3/2}\left(\sqrt{(a\alpha)^2+(b\beta)^2}\right)}{\left(\sqrt{(a\alpha)^2+(b\beta)^2}\right)^{3/2}}\ .$$

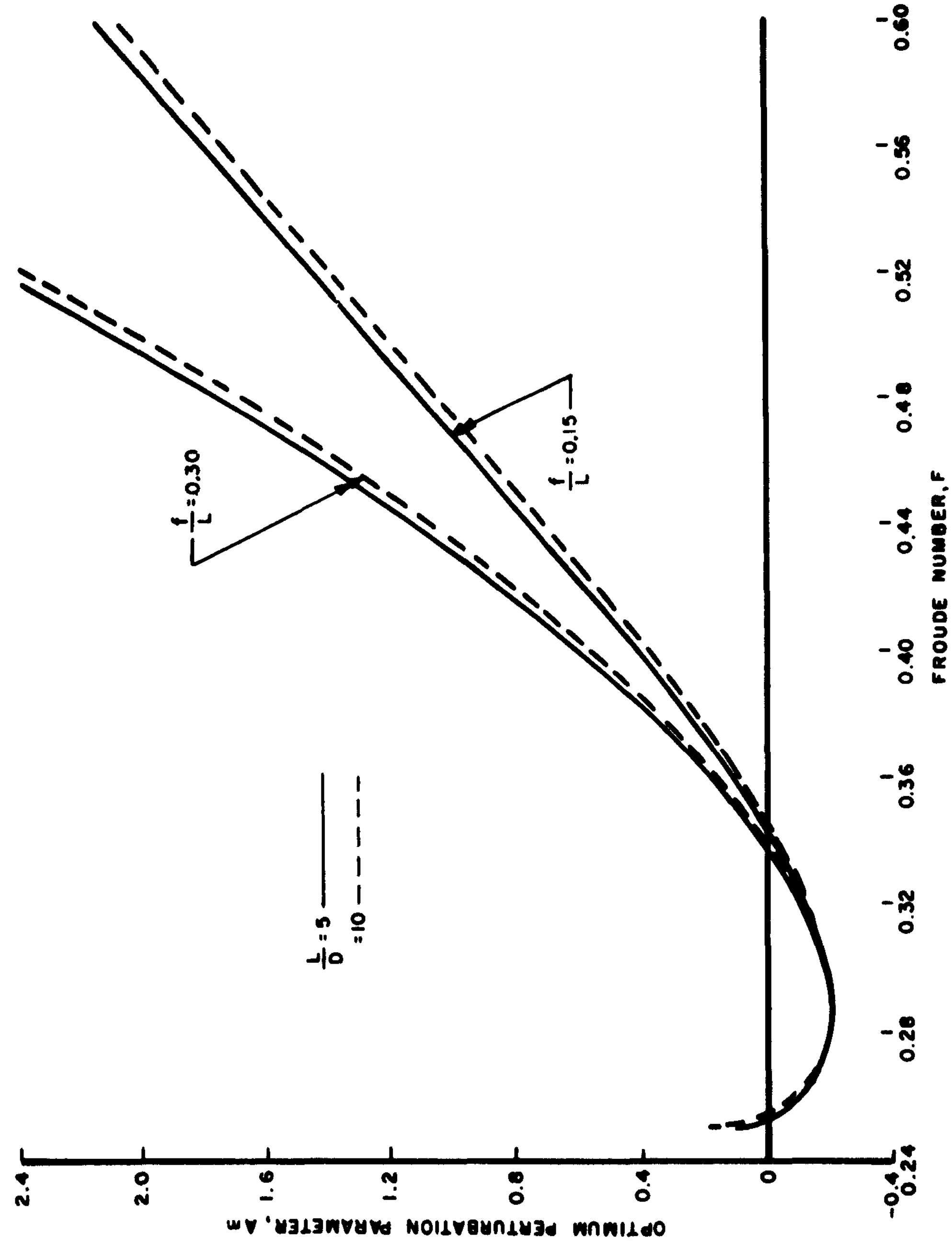

FIGURE I. OPTIMUM PERTURBATION PARAMETER VS FROUDE NUMBER.

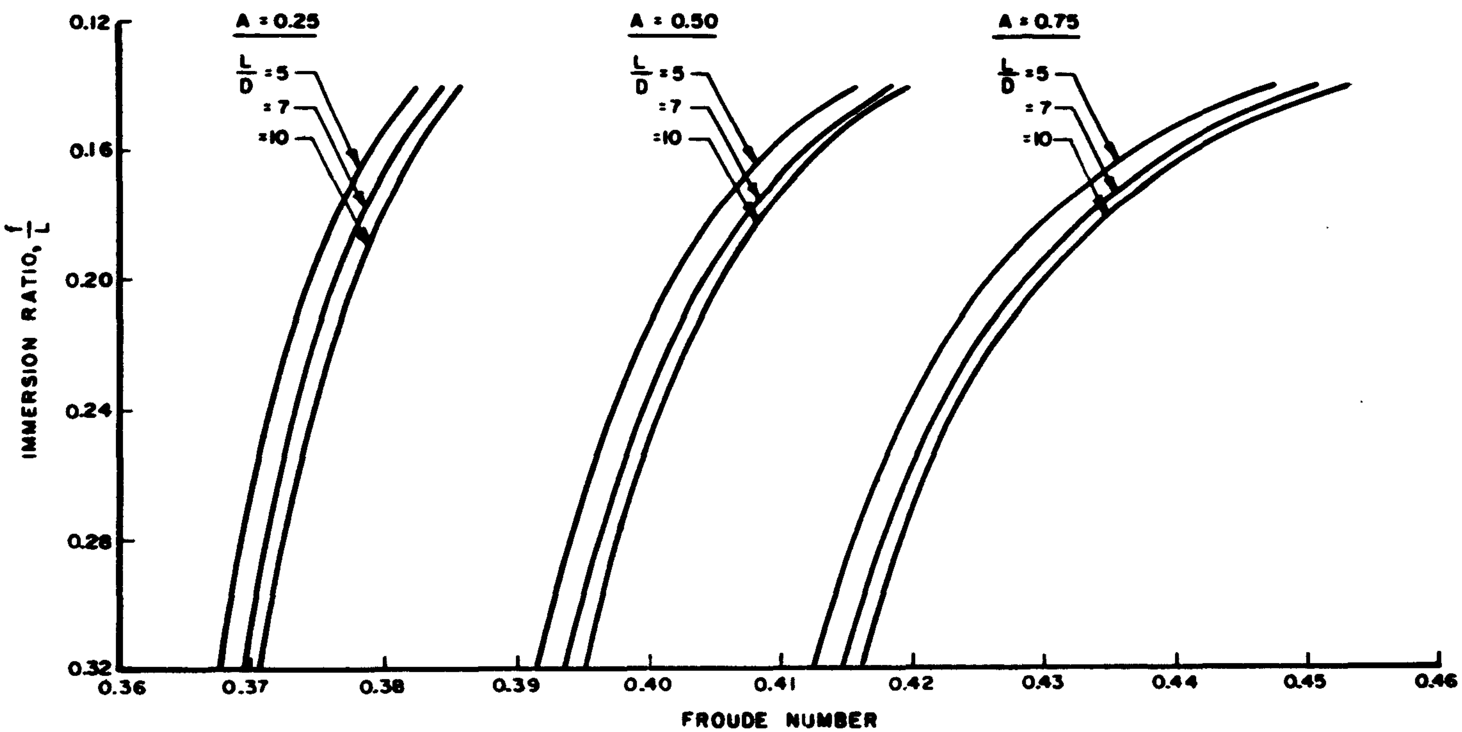

FIGURE 2. MINIMUM WAVE RESISTANCE OF THREE PERTURBED SPHEROID AS A FUNCTION OF FROUDE NUMBER AND IMMERSION RATIO.

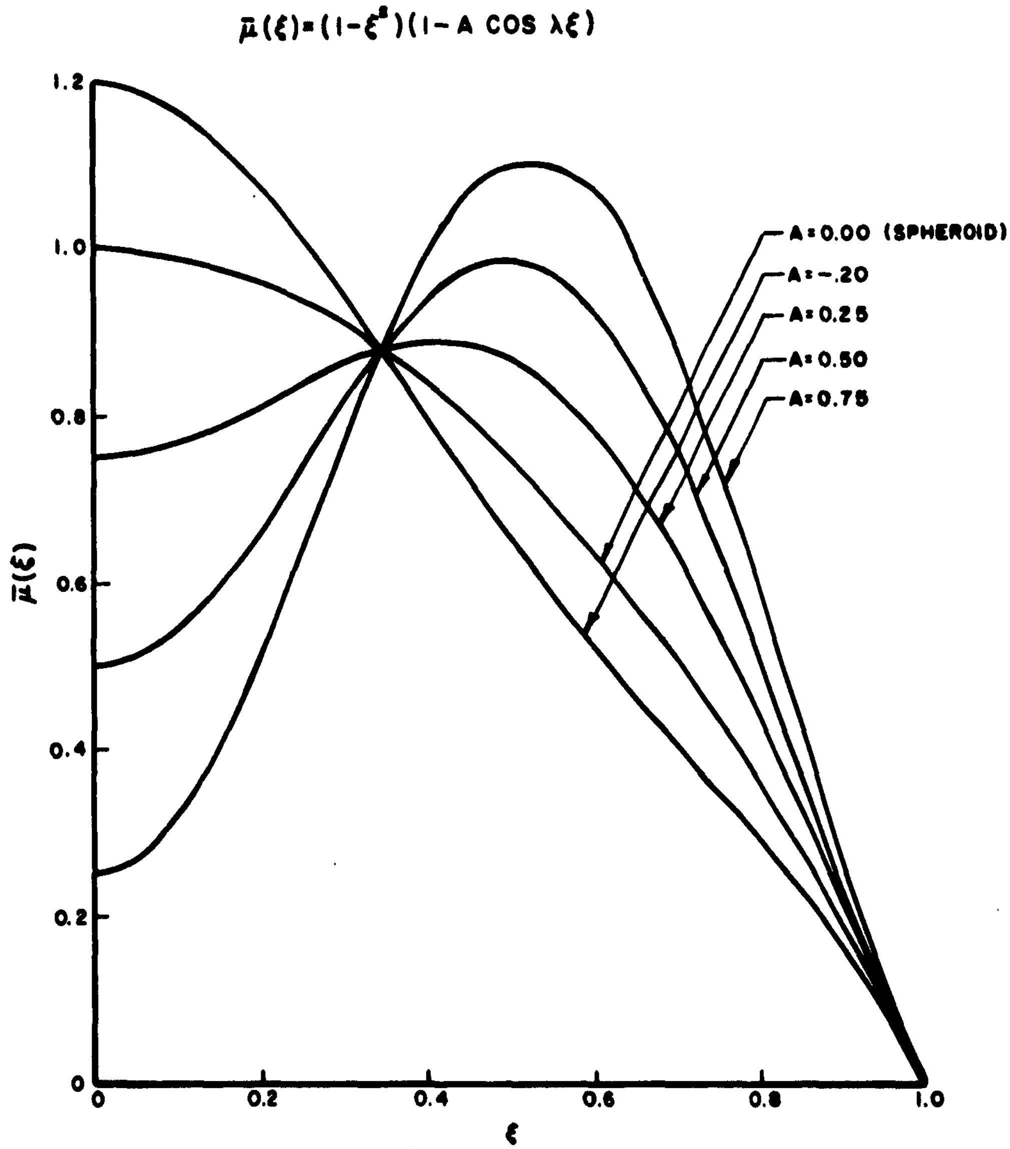

FIGURE 3. NORMALIZED DOUBLET DISTRIBUTION, $\bar{\mu}(\xi)$.

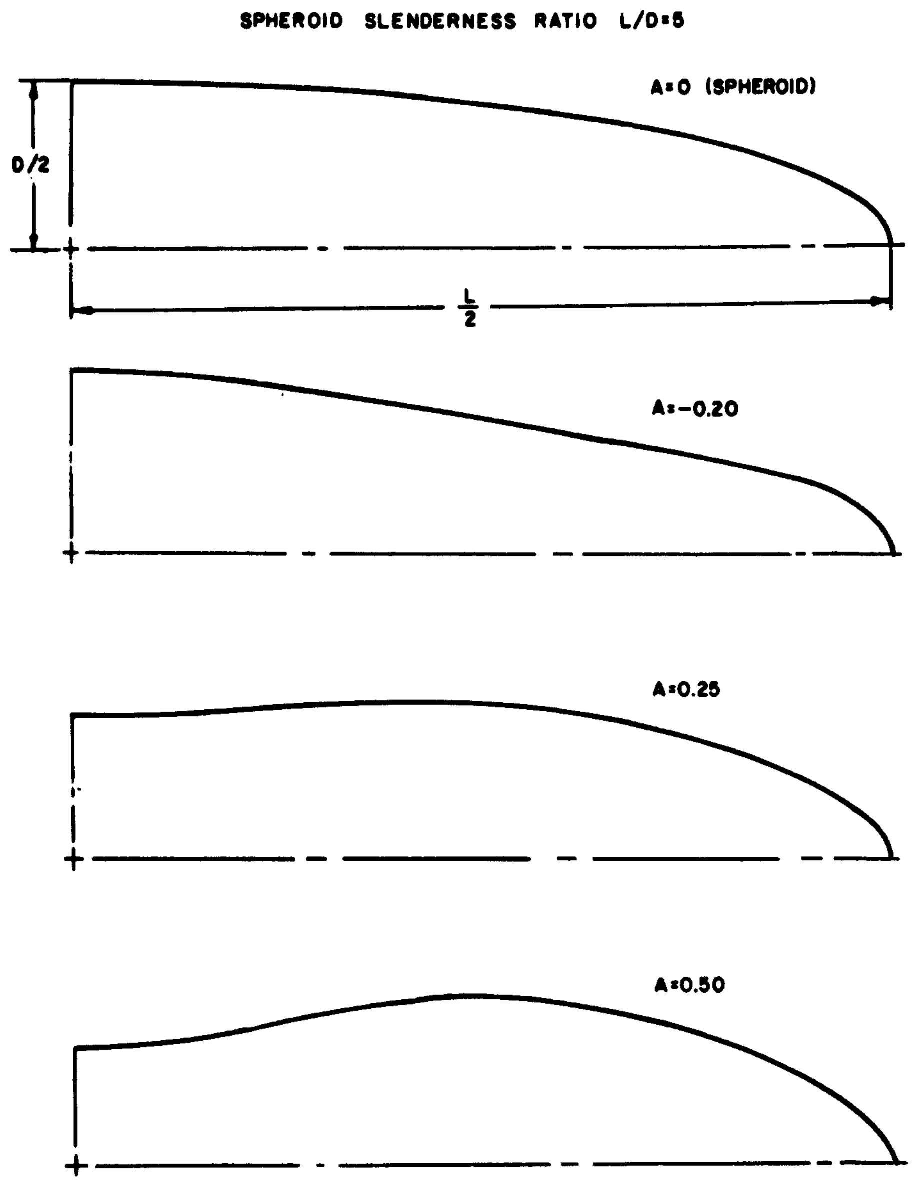

FIGURE 4. CONFIGURATION OF PERTURBED SPHEROID

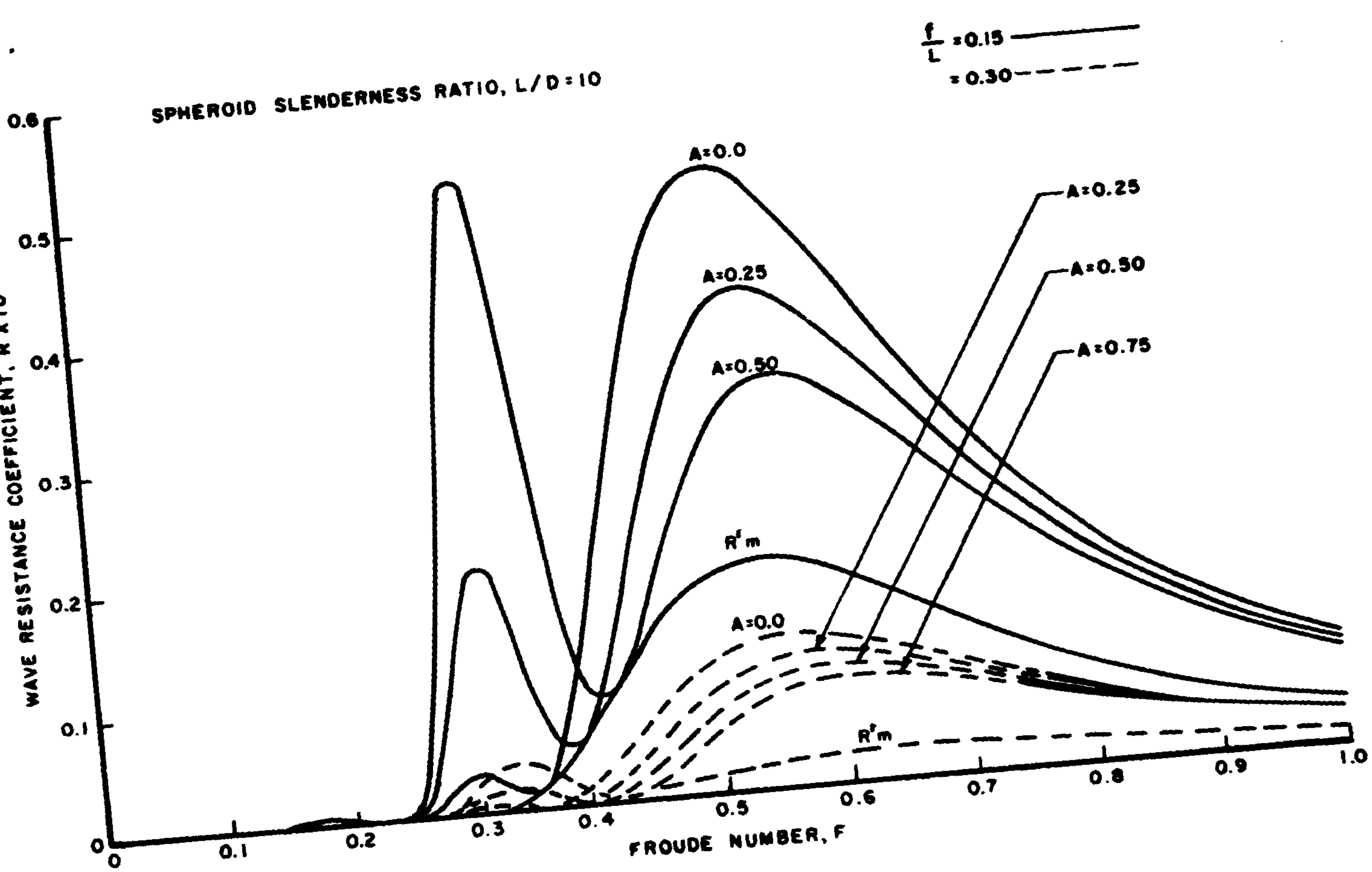

FIGURE 5. WAVE RESISTANCE COEFFICIENT OF PERTURBED SPHEROID VS FROUDE NUMBER.

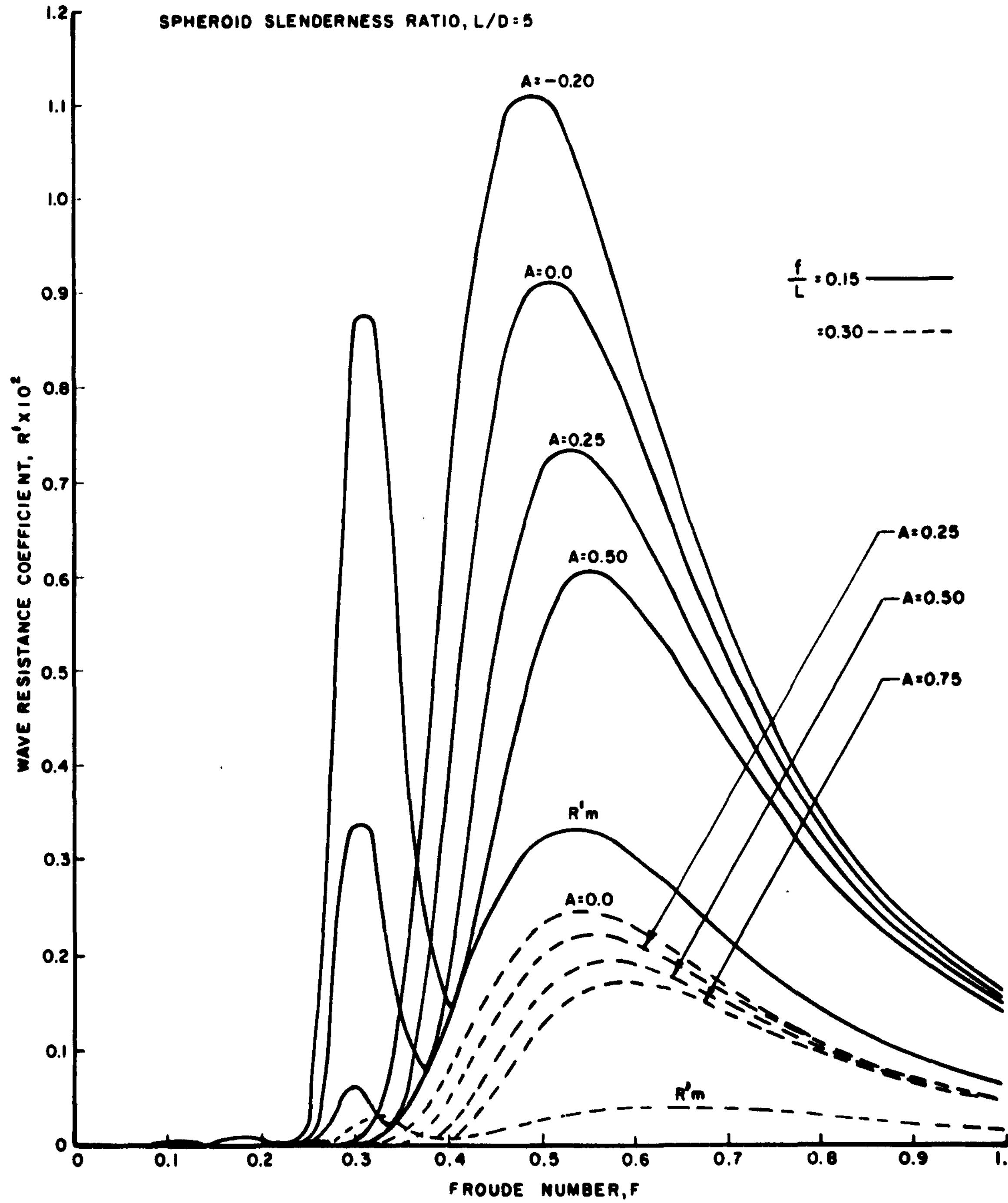

FIGURE 6. WAVE RESISTANCE COEFFICIENT OF PERTURBED SPHEROID VS FROUDE NUMBER.

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WAVE-RESISTANCE REDUCTION OF NEAR-SURFACE BODIES

Davidson Laboratory Report No. 933
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by King Eng and Pung N. Hu, March 1963

An analytical study of the wave-resistance characteristics of near-surface bodies was conducted to determine 1) for a given length and displacement, what changes in body-surface geometry are necessary to cause wave-resistance reduction, and 2) how geometrical change affects the wave-resistance behavior with Froude number and submergence depth. The general wave-resistance expression for a perturbed ellipsoid with the constraints of constant displacement and length is formulated. A digital computer solution of this variational problem is obtained for the case of the spheroid due to available computer-size limitations.

The effects of fineness ratio and submergence-to-length ratio on the Froude number behavior of the wave resistance for a range of perturbations is demonstrated. Substantial reduction in wave resistance is possible for all Froude numbers above and slightly below the optimum Froude number for a particular perturbation distribution. For Froude numbers lower than approximately 10% below the optimum Froude number, a large increase in wave-resistance coefficient may be obtained depending upon the perturbation used. Since this generally occurs at low Froude numbers, the actual increase in total resistance experienced for perturbations yielding acceptable geometrical changes should be quite acceptable. Depending upon the optimum Froude number, the geometrical changes required for wave-resistance reduction fall into two classes: 1) midsection bulge with finer bow and stern for Froude numbers below 0.32; and 2) above 0.32 Froude number a midsection pinch with bulging bow and stern.

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